Topic: Value that makes two tangent lines parallel

Question: What is the value of a such that the tangent lines of the function at $x = a$ and $x = a + 1$ are parallel?

$$f(x) = x^3 + x^2 + x - 1$$

Answer choices:

A $\frac{5}{6}$

B $-\frac{1}{2}$

C $-\frac{5}{6}$

D $\frac{1}{2}$

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Solution: C

When lines are parallel, it means their slopes are equal. To solve for the value that makes two tangent lines parallel, use the equation for a tangent line

$$y = f(a) + f'(a)(x - a)$$

where $f(a)$ is the given function at $x = a$, and

$$y = f(a + 1) + f'(a + 1)(x - (a + 1))$$

where $f(a + 1)$ is the given function at $x = a + 1$.

We need to find all the pieces in this tangent line equation, so we'll start with $f(a)$.

$$f(x) = x^3 + x^2 + x - 1$$

$$f(a) = a^3 + a^2 + a - 1$$

Find $f'(a)$.

$$f(x) = x^3 + x^2 + x - 1$$

$$f'(x) = 3x^2 + 2x + 1$$

$$f'(a) = 3a^2 + 2a + 1$$

These are the values for the tangent line at a. Now we need to find the same values for $a + 1$. We'll start with $f(a + 1)$.

$$f(x) = x^3 + x^2 + x - 1$$

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$$f(a+1) = (a+1)^3 + (a+1)^2 + (a+1) - 1$$

$$f(a+1) = \left[(a+1)\left(a^2+2a+1\right)\right] + a^2 + 2a + 1 + a + 1 - 1$$

$$f(a+1) = a^3 + 2a^2 + a + a^2 + 2a + 1 + a^2 + 2a + 1 + a + 1 - 1$$

$$f(a+1) = a^3 + 4a^2 + 6a + 2$$

Find $f'(a+1)$.

$$f(x) = x^3 + x^2 + x - 1$$

$$f'(x) = 3x^2 + 2x + 1$$

$$f'(a+1) = 3(a+1)^2 + 2(a+1) + 1$$

$$f'(a+1) = 3\left(a^2+2a+1\right) + 2a + 2 + 1$$

$$f'(a+1) = 3a^2 + 6a + 3 + 2a + 2 + 1$$

$$f'(a+1) = 3a^2 + 8a + 6$$

Now we can plug all the information we've found for a and $a+1$ into the tangent line formula.

At $x = a$,

$$y = f(a) + f'(a)(x-a)$$

$$y = \left(a^3 + a^2 + a - 1\right) + \left(3a^2 + 2a + 1\right)(x-a)$$

$$y = a^3 + a^2 + a - 1 + 3a^2x - 3a^3 + 2ax - 2a^2 + x - a$$

$$y = 3a^2x + 2ax + x - 2a^3 - a^2 - 1$$

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$$y = \left(3a^2 + 2a + 1\right)x - 2a^3 - a^2 - 1$$

At $a + 1$,

$$y = f(a+1) + f'(a+1)(x - (a+1))$$

$$y = \left(a^3 + 4a^2 + 6a + 2\right) + \left(3a^2 + 8a + 6\right)(x - a - 1)$$

$$y = a^3 + 4a^2 + 6a + 2 + 3a^2x - 3a^3 - 3a^2 + 8ax - 8a^2 - 8a + 6x - 6a - 6$$

$$y = 3a^2x + 8ax + 6x - 2a^3 - 7a^2 - 8a - 4$$

$$y = \left(3a^2 + 8a + 6\right)x - 2a^3 - 7a^2 - 8a - 4$$

In the slope-intercept formula for the equation of a line $y = mx + b$, m represents the slope of the line. Because we're looking to make these lines parallel, and that means their slopes must be equal to each other, we need to set the slopes equal to one another.

$$3a^2 + 2a + 1 = 3a^2 + 8a + 6$$

$$2a + 1 = 8a + 6$$

$$-6a = 5$$

$$a = -\frac{5}{6}$$

This is the value of a that makes the tangent lines parallel.

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Topic: Value that makes two tangent lines parallel

Question: What is the value of a such that the tangent lines of the function at $x = a$ and $x = a + 1$ are parallel?

$$f(x) = 2x^3 - x^2 + x + 12$$

Answer choices:

A $\frac{1}{3}$

B $-\frac{1}{3}$

C $-\frac{1}{2}$

D $\frac{1}{2}$

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Solution: B

When lines are parallel, it means their slopes are equal. To solve for the value that makes two tangent lines parallel, use the equation for a tangent line

$$y = f(a) + f'(a)(x - a)$$

where $f(a)$ is the given function at $x = a$, and

$$y = f(a + 1) + f'(a + 1)(x - (a + 1))$$

where $f(a + 1)$ is the given function at $x = a + 1$.

We need to find all the pieces in this tangent line equation, so we'll start with $f(a)$.

$$f(x) = 2x^3 - x^2 + x + 12$$

$$f(a) = 2a^3 - a^2 + a + 12$$

Find $f'(a)$.

$$f(x) = 2x^3 - x^2 + x + 12$$

$$f'(x) = 6x^2 - 2x + 1$$

$$f'(a) = 6a^2 - 2a + 1$$

These are the values for the tangent line at a. Now we need to find the same values for $a + 1$. We'll start with $f(a + 1)$.

$$f(x) = 2x^3 - x^2 + x + 12$$

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$$f(a+1) = 2(a+1)^3 - (a+1)^2 + (a+1) + 12$$

$$f(a+1) = 2\left[(a+1)\left(a^2+2a+1\right)\right] - a^2 - 2a - 1 + a + 1 + 12$$

$$f(a+1) = 2\left(a^3+2a^2+a+a^2+2a+1\right) - a^2 - 2a - 1 + a + 1 + 12$$

$$f(a+1) = 2a^3 + 4a^2 + 2a + 2a^2 + 4a + 2 - a^2 - 2a - 1 + a + 1 + 12$$

$$f(a+1) = 2a^3 + 5a^2 + 5a + 14$$

Find $f'(a+1)$.

$$f(x) = 2x^3 - x^2 + x + 12$$

$$f'(x) = 6x^2 - 2x + 1$$

$$f'(a+1) = 6(a+1)^2 - 2(a+1) + 1$$

$$f'(a+1) = 6\left(a^2+2a+1\right) - 2(a+1) + 1$$

$$f'(a+1) = 6a^2 + 12a + 6 - 2a - 2 + 1$$

$$f'(a+1) = 6a^2 + 10a + 5$$

Now we can plug all the information we've found for a and $a+1$ into the tangent line formula.

At $x = a$,

$$y = f(a) + f'(a)(x-a)$$

$$y = \left(2a^3 - a^2 + a + 12\right) + \left(6a^2 - 2a + 1\right)(x-a)$$

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$$y = 2a^3 - a^2 + a + 12 + 6a^2x - 2ax + x - 6a^3 + 2a^2 - a$$

$$y = 6a^2x - 2ax + x - 4a^3 + a^2 + 12$$

$$y = \left(6a^2 - 2a + 1\right)x - 4a^3 + a^2 + 12$$

At $a + 1$,

$$y = f(a+1) + f'(a+1)(x - (a+1))$$

$$y = \left(2a^3 + 5a^2 + 5a + 14\right) + \left(6a^2 + 10a + 5\right)(x - a - 1)$$

$$y = 2a^3 + 5a^2 + 5a + 14 + 6a^2x - 6a^3 - 6a^2 + 10ax - 10a^2 - 10a + 5x - 5a - 5$$

$$y = 6a^2x + 10ax + 5x - 4a^3 - 11a^2 - 10a + 9$$

$$y = \left(6a^2 + 10a + 5\right)x - 4a^3 - 11a^2 - 10a + 9$$

In the slope-intercept formula for the equation of a line $y = mx + b$, m represents the slope of the line. Because we're looking to make these lines parallel, and that means their slopes must be equal to each other, we need to set the slopes equal to one another.

$$6a^2 - 2a + 1 = 6a^2 + 10a + 5$$

$$-2a + 1 = 10a + 5$$

$$-12a = 4$$

$$a = -\frac{1}{3}$$

This is the value of a that makes the tangent lines parallel.

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Topic: Value that makes two tangent lines parallel

Question: What is the value of a such that the tangent lines of the function at $x = a$ and $x = a + 2$ are parallel?

$$f(x) = x^3 + 3x^2 - x - 5$$

Answer choices:

A $\frac{1}{2}$

B $-\frac{1}{2}$

C 2

D -2

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Solution: D

When lines are parallel, it means their slopes are equal. To solve for the value that makes two tangent lines parallel, use the equation for a tangent line

$$y = f(a) + f'(a)(x - a)$$

where $f(a)$ is the given function at $x = a$, and

$$y = f(a + 2) + f'(a + 2)(x - (a + 2))$$

where $f(a + 2)$ is the given function at $x = a + 2$.

We need to find all the pieces in this tangent line equation, so we'll start with $f(a)$.

$$f(x) = x^3 + 3x^2 - x - 5$$

$$f(a) = a^3 + 3a^2 - a - 5$$

Find $f'(a)$.

$$f(x) = x^3 + 3x^2 - x - 5$$

$$f'(x) = 3x^2 + 6x - 1$$

$$f'(a) = 3a^2 + 6a - 1$$

These are the values for the tangent line at a. Now we need to find the same values for $a + 2$. We'll start with $f(a + 2)$.

$$f(x) = x^3 + 3x^2 - x - 5$$

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$$f(a+2) = (a+2)^3 + 3(a+2)^2 - (a+2) - 5$$

$$f(a+2) = (a+2)\left(a^2+4a+4\right) + 3\left(a^2+4a+4\right) - a - 2 - 5$$

$$f(a+2) = a^3 + 4a^2 + 4a + 2a^2 + 8a + 8 + 3a^2 + 12a + 12 - a - 2 - 5$$

$$f(a+2) = a^3 + 9a^2 + 23a + 13$$

Find $f'(a+2)$.

$$f(x) = x^3 + 3x^2 - x - 5$$

$$f'(x) = 3x^2 + 6x - 1$$

$$f'(a+2) = 3(a+2)^2 + 6(a+2) - 1$$

$$f'(a+2) = 3\left(a^2+4a+4\right) + 6a + 12 - 1$$

$$f'(a+2) = 3a^2 + 12a + 12 + 6a + 12 - 1$$

$$f'(a+2) = 3a^2 + 18a + 23$$

Now we can plug all the information we've found for a and $a+2$ into the tangent line formula.

At $x = a$,

$$y = f(a) + f'(a)(x-a)$$

$$y = \left(a^3 + 3a^2 - a - 5\right) + \left(3a^2 + 6a - 1\right)(x-a)$$

$$y = a^3 + 3a^2 - a - 5 + 3a^2x + 6ax - x - 3a^3 - 6a^2 + a$$

$$y = 3a^2x + 6ax - x - 2a^3 - 3a^2 - 5$$

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$$y = \left(3a^2 + 6a - 1\right)x - 2a^3 - 3a^2 - 5$$

At $a + 2$,

$$y = f(a+2) + f'(a+2)(x - (a+2))$$

$$y = \left(a^3 + 9a^2 + 23a + 13\right) + \left(3a^2 + 18a + 23\right)(x - a - 2))$$

$$y = a^3 + 9a^2 + 23a + 13 + 3a^2x - 3a^3 - 6a^2 + 18ax - 18a^2 - 36a + 23x - 23a - 46$$

$$y = 3a^2x + 18ax + 23x - 2a^3 - 15a^2 - 36a - 33$$

$$y = \left(3a^2 + 18a + 23\right)x - 2a^3 - 15a^2 - 36a - 33$$

In the slope-intercept formula for the equation of a line $y = mx + b$, m represents the slope of the line. Because we're looking to make these lines parallel, and that means their slopes must be equal to each other, we need to set the slopes equal to one another.

$$3a^2 + 6a - 1 = 3a^2 + 18a + 23$$

$$6a - 1 = 18a + 23$$

$$-12a = 24$$

$$a = -2$$

This is the value of a that makes the tangent lines parallel.

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